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Conductance of a large point contact with Rashba effect

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Abstract. We study the scattering of an electron of a 2DEG through a large point contact separating a region where the electrons are free and a region where the Rashba spin-orbit coupling is present. The scattering depends dramatically on the electron incidence angle showing double refraction within the Rashba region. For incidence not normal to the interface the electron spin state is not conserved. The calculated conductance exhibits an oscillating behavior as a function of spin state of the incident electrons with different spin down and spin up currents. Our model describes both a ferromagnetic semimetallic source and a simple metallic injection electrode. In the first case the electrons are injected in a pure spin state and in the second one they are unpolarized, that is in a statistical mixture of spin up and down states. In both the cases the passage through the large point contact produces spin polarized currents.

PACS. 85.75.Hh Spin polarized field effect transistors – 72.25.-b Spin polarized transport – 73.23.Ad Ballistic transport

1 Introduction

One of the problems in the field of spintronic is to build a source of spin-polarized electrons [1,2]. Several devices based on the Rashba [3] effect have been proposed in the last decade. They start from the pioneering work of Datta and Das [4] that proposed an electronic equivalent of an electro-optical modulator. The idea is to consider a two dimensional electron gas (2DEG) with a spin-orbit coupling (Rashba effect) due to an electrical field perpendicular to the plane in which the electrons are contained. In this confined system the electronic spin behaves like the polarization of the light in a biaxial crystal.

The Datta and Das ideas have inspired some investigations on spintronic devices that exhibit spin-valves effects [5,6]. In particular, the transport through a single interface ferromagnet-2DEG was considered claiming for an oscillatory spin-filtering due to a spin-dependent conductance [7]. However there are same intrinsic obstacles to use this technique due mainly to the conductivity mismatch between metals and semiconductors [8]. Some devices that achieve spin filtering without using ferromagnets have been proposed. We mention among the others a mesoscopic Stern-Gerlach interferometric device based on non dispersive phases (Aharonov-Bohm and Rashba) [9] and a pair of quantum wires tunnel-coupled under Rashba spin-orbit interaction [10]. The attempt to avoid ferromagnets is the main aim of our study. We show that a spin-dependent conductance can be achieved by using large point contact and spin-unpolarized electrons. In order to support our claim, we present a detailed study of the scattering that an electron in a 2DEG undergoes when it passes from a region without the spin-orbit coupling ("no Rashba" NR zone) to a region in which the spin-orbit coupling is present ("Rashba" R zone). We start on considering the electron in a pure spin state fixed by the magnetization of a semimetallic ferromagnetic lead, then we apply our results to the case of unpolarized electrons injected by a metallic lead, that is in a statistical mixture of spin up and spin down. We focus our attention on the scattering with an incidence angle not orthogonal to the interface since it is expected to give spin dependent contributions to the conductance of a large point contact. Two different spin-polarized output channels appear. The behaviour of electron spin in such scattering can be compared with the polarization of the light in a biaxial crystal: the incident ray splits, within the crystal, in two rays (ordinary and extraordinary) whose polarizations are orthogonal. The electron motion within the hybrid system is assumed to be ballistic and the conductance of a wide Point Contact separating the NR and the R zones can be calculated by summing up the transmission coefficients obtained varying the allowed incidence angles from 0 (normal incidence) to limit angles at which the two output spin channels are completely reflected. We show that the conductance is made by different spin up and spin down contributions and depends on the spin state of the

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incoming electrons. The injection of electrons in the Rashba zone through the Point Contact is a way to spin-polarize the 2DEG electrons because the output spin up current is different from the spin down current.

The paper is organized in the following way. In the second section we resume the properties of a 2DEG with Rashba coupling everywhere in the plane. In the third one we analyze the scattering on the interface between the NR and the R zone. In the last section we present the calculation of the Point Contact conductance. Short conclusions end the paper. In the appendices we report the linear system of the wave amplitudes as stems out from the boundary conditions at the interface. In the last appendix we briefly classify the Rashba Hamiltonian symmetry.

2 2DEG with Rashba spin-orbit coupling everywhere

In this section we recall the characteristics of a 2DEG with a spin-orbit Rashba coupling occupying the whole $x-z$ plane [11]. An electrical field $\mathcal{E}\widehat{y}$ ($\widehat{y}$ is the unit vector in y direction) acts on the electrons in the plane and we assume that the electronic transport within the plane can be considered as ballistic. The velocities of the charge carriers are of the order 10^8 cm/s or larger and a magnetic field parallel to the plane appears in the rest reference frame of the charges. This kind of spin-orbit is known as Rashba effect. The Hamiltonian of the 2DEG is

$$H_0 = \frac{p^2}{2m_S} + H_{SO} \tag{1}$$

where H_{SO} is the spin-orbit term

$$H_{SO} = \frac{\eta}{\hbar}\,\widehat{y}\cdot(\vec{\sigma}\times\vec{p}). \tag{2}$$

We represent the spin $\vec{s} = \vec{\sigma}/2$ with the Pauli matrices $\vec{\sigma}$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and $\eta \propto \mathcal{E}$. The spin-orbit term in the Hamiltonian results in the spin splitting of free electron band (see Fig. 1). Indeed, as it has been demonstrated by Lommer et al. [12] for quasi two dimensional semiconducting quantum wells, such splitting appears even in the absence of magnetic field, due to the quantization in the confinement potential. For the InGaAs/InAlAs heterostructure the spin-orbit parameter η was estimated to be $\sim 3.9\times10^{-12}$ eV m [13]. Since H commutes with $\vec{p}$ we can classify the eigenvectors of H_0 with the wavenumbers k_x and k_z. The electron eigenstates corresponding to the splitted energy levels $E_\pm$ are the spinors

$$\psi_+ = \exp\left[i\left(k_x x + k_z z\right)\right]\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$$

$$\psi_- = \exp\left[i\left(k_x x + k_z z\right)\right]\begin{pmatrix}-\sin\theta \\ \cos\theta\end{pmatrix} \tag{3}$$

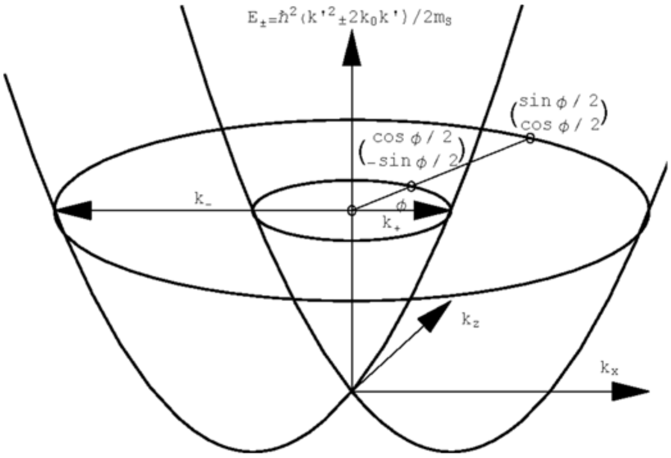

Fig. 1. Free electron spin splitted bands by spin-orbit Rashba effect. The two circles of radii k_+ and k_- represent constant energy contours. The spinor parts of the wave functions are indicated.

whose eigenvalues are

$$E_\pm = \frac{\hbar^2}{2m_S}\left(k_x^2 + k_z^2\right) \pm \eta\sqrt{k_x^2 + k_z^2}. \tag{4}$$

Here

$$\theta = \arctan\left[\frac{k_x}{k_z} - \sqrt{\frac{k_x^2}{k_z^2} + 1}\right]. \tag{5}$$

If $k' = \sqrt{k_z^2 + k_z^2}$ is the modulus of the momentum, and

$$\phi = \arctan\frac{k_z}{k_x}$$

its direction in the plane, then

$$\theta = -\frac{\phi}{2} \tag{6}$$

and

$$E_\pm = \frac{\hbar^2}{2m_S}\left(k'^{\,2} \pm 2k_0 k'\right) \qquad (k_0 = m_S\eta/\hbar^2). \tag{7}$$

One can see that the spin degeneracy on the Fermi surface is lifted but the Rashba term is not able to produce a spontaneous spin polarization of the electron states. For given energy there are two different values of k' with any spin projection. The meaning of equation (6) is: when we choose the direction of electron motion fixing its k_x and k_z, then we assign automatically the electron spin polarization state. If $\vec{k}'$ is directed along x then $\phi = 0$ and ψ_+, ψ_- describe the pure "spin up" and "spin down" states in z direction, that we fix as the spin quantization direction. We note that the account for the spin-orbit interaction in the Hamiltonian (1) reduces the rank of the direct space group twice [14]: space rotation of 4π is needed to get the same spinor.

Let us denote the complex conjugation operator as $\widehat{K}_0$:

$$\widehat{K}_0 f = f^*.$$

The time reversal operator [15] for the special case of a particle of spin $\frac{1}{2}$ takes the form

$$\widehat{K} = -i\sigma_y \widehat{K}_0$$

and

$$\widehat{K}\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} -f_2^* \\ f_1^* \end{pmatrix}. \quad (8)$$

It commutes [15] with H_{SO}. Applying $\widehat{K}$ to the degenerate eigenstates ψ_+, ψ_- we see that one is the time reversed of the other

$$\widehat{K}\psi_+ = -\psi_-^*, \quad \widehat{K}\psi_- = \psi_+^* \quad (9)$$

while their spinor parts s_+ and s_-

$$s_+ = \begin{pmatrix} \cos\phi/2 \\ -\sin\phi/2 \end{pmatrix}, \; s_- = \begin{pmatrix} \sin\phi/2 \\ \cos\phi/2 \end{pmatrix}$$

are one orthogonal to the other.

Finally we stress that the spin-orbit interaction can be attributed to a magnetic field parallel to the plane and orthogonal to the wave vector $\vec{k}'$. This magnetic field couples with the spin magnetic moment and it orientates the spin along the direction orthogonal to $\vec{k}'$ [3]. The spin component in this direction is

$$\sigma_\perp = -\sin\phi \cdot \sigma_x + \cos\phi \cdot \sigma_z = \begin{pmatrix} \cos\phi & -\sin\phi \\ -\sin\phi & -\cos\phi \end{pmatrix}$$

and s_+ and s_- are eigenstates of $\sigma_\perp$

$$\sigma_\perp s_+ = s_+ \; ; \quad \sigma_\perp s_- = -s_-.$$

3 Scattering at an interface with a seminfinite Rashba 2DEG

We assume that on the $x-z$ plane the Rashba coupling is restricted to $x > 0$ region and an electron with a momentum $\vec{k} \equiv (k\cos\gamma, k\sin\gamma)$ and an energy $E = \hbar^2 k^2/2m_F$ is incoming in the pure spin state $|\delta\rangle$ from the NR zone (where the electron mass is m_F). The pure spin state we use is

$$|\delta\rangle = \cos\delta\,|\uparrow\rangle + \sin\delta\,|\downarrow\rangle$$

where the ket $|\uparrow\rangle$ indicates the spin up state with $s_z = 1/2$ and $|\downarrow\rangle$ is the spin down state with $s_z = -1/2$. The incident wave function is

$$\psi_i = \exp\left(ik\left(x\cos\gamma + z\sin\gamma\right)\right)|\delta\rangle$$

while the reflected wave function is

$$\psi_r = \exp\left(ik\left(-x\cos\gamma + z\sin\gamma\right)\right)\left(r_\uparrow\,|\uparrow\rangle + r_\downarrow\,|\downarrow\rangle\right).$$

We have in the output, within the R zone ($x > 0$), a superposition of the two states of the spin splitted bands $E_\pm(k')$

$$E_\pm(k') = \hbar^2\left(k'^2 \pm 2k_0 k'\right)/2m_S$$

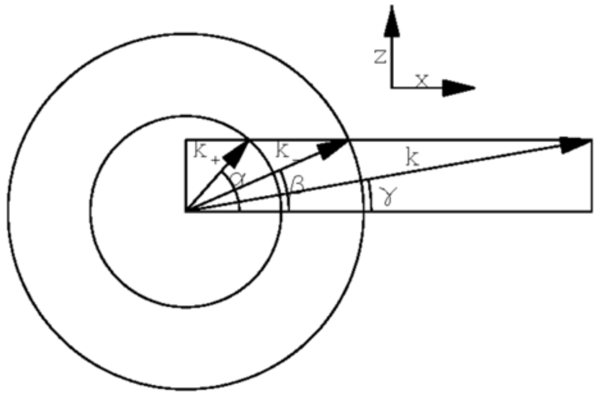

Fig. 2. The vectors $\vec{k}_+$, $\vec{k}_-$, $\vec{k}$ in $k-$space and the angles α, β and γ that they form with x direction normal to the interface. The two circles are the lines at the constant energy $\hbar^2 k^2/2m_F$.

degenerate with the same energy E. The energy conservation fixes two values for the modulus of the wave vector k', and from

$$E_\pm(k') \equiv E = \hbar^2 k^2/2m_F,$$

we get

$$k' = \sqrt{\mu k^2 + k_0^2} \mp k_0 = k_\pm \quad (\mu = m_S/m_F).$$

The directions of $\vec{k}_+$ and $\vec{k}_-$ are fixed by the conservation of the momentum parallel to the interface

$$k_{+,x} = k_{-,x} = k_x.$$

Now the angle ϕ for the mode + takes a value α different from its value β for the mode $-$. The angle α of $\vec{k}_+$, the angle β of $\vec{k}_-$ and γ of $\vec{k}$ with the x axis are linked up by the relationship

$$k_+\sin\alpha = k_-\sin\beta = k\sin\gamma.$$

The transmitted wave function at $x > 0$ is the superposition of the transmitted ones in both the modes (+) and ($-$)

$$\psi_t = t_+\exp\left(ik_+\left(x\cos\alpha + z\sin\alpha\right)\right)\begin{pmatrix} \cos\alpha/2 \\ -\sin\alpha/2 \end{pmatrix} + t_-\exp\left(ik_-\left(x\cos\beta + z\sin\beta\right)\right)\begin{pmatrix} \sin\beta/2 \\ \cos\beta/2 \end{pmatrix}. \quad (10)$$

Figure 2 shows the output angles α and β. The two modes have the same energy E along the two circles. The conservation of k_z gives α and β as functions of the incidence angle γ. Only when the incidence is normal, with $\gamma = 0$, the outgoing wave functions (+) and ($-$) go in the same direction with $\alpha = \beta = 0$ and with the two different wave vectors k_+ and k_-. In the other cases they go along two different directions. We are facing with a

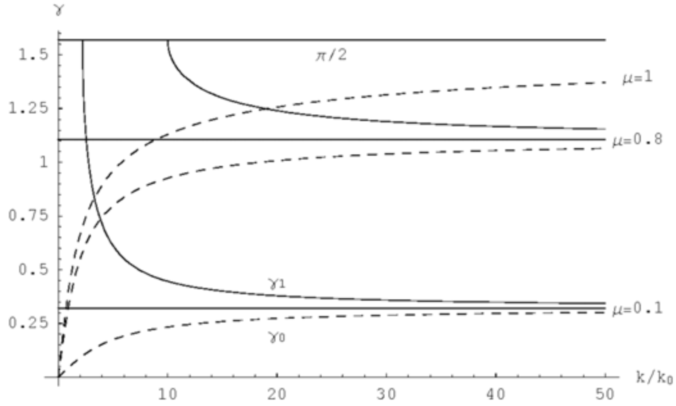

Fig. 3. The limit angles γ_0 of + mode (dashed line) and γ_1 of − mode (full line), for three different values of mass ratio μ, as functions of k/k_0. For γ above γ_1 the total reflection occurs.

phenomenon analogous to the double refraction that appears in biaxial crystals [16] with two outgoing divergent rays. The birefringence arises when the characteristics of electromagnetic propagation depend on the directions of propagation and polarization of the light wave. In our case the spin of the electron wave functions behaves like the polarization of the light. We notice that the spin orientations of the outgoing waves (+) and (−) are fixed by the output angles α and β according to the equation (6). The crossing of the interface changes the spin state. The electron exits in the R zone in a superposition of the two spin states

$$\begin{pmatrix} \cos\alpha/2 \\ -\sin\alpha/2 \end{pmatrix} \text{ and } \begin{pmatrix} \sin\beta/2 \\ \cos\beta/2 \end{pmatrix}.$$

The output angles α and β are functions of k, γ, k_0 and μ and they do not depend on the incident spin orientation angle δ.

The mode (+) has the limit angle

$$\gamma_0 = \arcsin\frac{k_+}{k} \tag{11}$$

and, for $\gamma > \gamma_0$, this mode is totally reflected and it vanishes exponentially for $x > 0$. Here and in the following we take $0 < \mu < 1$: we assume that the effective mass in the R zone is less than the effective mass in the injection electrode in NR zone. When $k/k_0 < 2/(1-\mu)$, the mode (−) is always transmitted up to grazing incidence at $\gamma = \pi/2$. Increasing the kinetic energy with respect to spin-orbit coupling when $k/k_0 > 2/(1-\mu)$, a second limit angle appears

$$\gamma_1 = \arcsin\frac{k_-}{k} > \gamma_0 \tag{12}$$

and for $\gamma > \gamma_1$ we have the total reflection (both the modes vanish for $x > 0$). When the strength of spin-orbit coupling goes to zero, γ_0 and γ_1 tend to the common limit $\arcsin\sqrt{\mu}$: lighter is the effective mass within the 2DEG nearer to the normal are the propagation directions $\alpha < \gamma_0$ and $\beta < \gamma_1$ allowed into R zone. Figure 3 shows the limit angles as a function of k/k_0. We note that when $\gamma > \gamma_0$ then

$$\sin\alpha = \frac{k}{k_+}\sin\gamma > 1$$

and α becomes complex

$$\alpha = \frac{\pi}{2} - i\alpha'.$$

The correct determination for its imaginary part $-\alpha'$ is obtained when $\alpha' > 0$ because

$$\sin\alpha = \cosh\alpha'; \;\; \cos\alpha = i\sinh\alpha'.$$

The mode (+) becomes a vanishing wave decaying along x axis while it is a propagating wave along z direction

$$\exp\left(-k_+ x\sinh\alpha'\right)\exp\left(ik_+ z\cosh\alpha'\right) \times \begin{pmatrix} \cos\left(\pi/4 - i\alpha'/2\right) \\ -\sin\left(\pi/4 - i\alpha'/2\right) \end{pmatrix}.$$

When $\gamma > \gamma_1$, $\beta = \pi/2 - i\beta'$ and both the modes are damped within the 2DEG: the incident wave is totally reflected.

To calculate the transmitted amplitudes t_+ and t_- in the (+) and (−) modes we introduce the hybrid system Hamiltonian

$$H = \vec{p}\frac{1}{2m(x)}\vec{p} + \frac{\eta(x)}{\hbar}\left(\sigma_z p_x - \sigma_x p_z\right) - i\sigma_z\frac{1}{2}\frac{\partial\eta(x)}{\partial x} + U\delta(x). \tag{13}$$

We assume that the mass and the strength of spin-orbit coupling are piecewise constant

$$1/m(x) = \vartheta(-x)/m_F + \vartheta(x)/m_S$$
$$\eta(x) = \eta\vartheta(x), \tag{14}$$

where $\vartheta(x)$ is the step function. The third term is needed to get an hermitian H operator and the fourth term regulates the transparency of the interface. The spinor eigenstate ψ of H is continuous while its derivative has a discontinuity fixed by the strength $u - i\sigma_z k_0$ of the Dirac delta in $x = 0$

$$\psi(0+) = \psi(0-)$$
$$\frac{\partial\psi(0+)}{\partial x} - \mu\frac{\partial\psi(0-)}{\partial x} = \left(u - ik_0\sigma_z\right)\psi(0). \tag{15}$$

This matching conditions give a four times four linear system for the amplitudes $t_+, t_-, r_\uparrow$ and $r_\downarrow$ that is reported in the Appendix A.

The normal incidence case deserves a special care [17,18]. When $\gamma = 0$ then $\alpha = \beta = 0$, the mode (+) is in the spin up state $|\uparrow\rangle$ while the mode (−) is in spin down state $|\downarrow\rangle$. In this case σ_z is a motion constant and a spin up $|\uparrow\rangle$ state goes entirely in $(+)$ mode being zero the amplitude transmitted in $(-)$ mode. A spin down state $|\downarrow\rangle$ goes entirely in $(-)$ mode with zero amplitude in $(+)$ mode. When $\gamma = 0$ with an incoming spin state $|\delta\rangle$ we have

$$t_+ = \frac{2\mu k \cos\delta}{k_+ + k_0 + iu + \mu k}, \qquad t_- = \frac{2\mu k \sin\delta}{k_- - k_0 + iu + \mu k}$$

but $k_+ + k_0 = k_- - k_0 = \sqrt{\mu k^2 + k_0^2}$ so that $t_+ = t_-$ and the transmitted spinor is

$$\psi_t(0-) = \frac{2\mu k}{\sqrt{\mu k^2 + k_0^2} + iu + \mu k} |\delta\rangle .$$

We stress that the passage of the interface do not change the spin. When $x > 0$ the spinor becomes $\exp(ik_+x)\cos\delta\,|\uparrow\rangle + \exp(ik_-x)\sin\delta\,|\downarrow\rangle$ and the propagation along a distance L into the Rashba region gives the phase difference on which is based the Datta and Das device. The inefficiency of the scattering at normal incidence to modify the spin state stems out from the identity

$$k_+ + k_0 = k_- - k_0$$

that comes from the following property of the Hamiltonian (13): changing the sign of k_0, the two modes $(+)$ and $(-)$ are interchanged one with the other. The symmetry of the Hamiltonian H is classified in the Appendix B. When $\gamma \neq 0$ the amplitudes $t_+, t_-, r_\uparrow$ and $r_\downarrow$ depend on k, γ, k_0, μ and on δ too, that is on the incoming spin state.

The square moduli of the transmitted amplitudes $|t_\pm(\delta)|^2$ are shown in Figure 4 when γ is between 0 and $\pi/2$. We see in Figure 4a that $|t_+(0)|^2$ and $|t_-(\pi/2)|^2$ start from the same value for $\gamma = 0$ but become different when the incidence angle increases towards $\pi/2$. In Figure 4b we report the same behavior for a different pair of orthogonal spins $\delta = \pi/4, 3\pi/4$. The derivatives of $|t_\pm(\delta)|^2$ jump in γ_0 and then in γ_1 when the character of the modes propagation changes. The cusps sign the limit angles.

In order to calculate the transmission coefficient T we need the probability current density

$$\vec{j} = \Re\left\{\psi^\dagger \vec{p}\psi\right\}; \quad x < 0$$

$$\vec{j} = \Re\left\{\psi^\dagger\left(\vec{p} + \hbar k_0 \cdot \widehat{y} \times \vec{\sigma}\right)\psi\right\}; \quad x > 0 \qquad (16)$$

whose $x-$components are

$$j_{xl} = \hbar k \cos\gamma\left(1 - |r_\uparrow|^2 - |r_\downarrow|^2\right)/m_F \quad \text{for } x < 0$$

$$j_{xr} = j_{x+} + j_{x-} \quad \text{for } x > 0 \quad \text{with}$$

$$j_{x+} = \hbar(k_+ + k_0)\cos\alpha \cdot |t_+|^2 / m_S$$

$$j_{x-} = \hbar(k_- - k_0)\cos\beta \cdot |t_-|^2 / m_S. \qquad (17)$$

The boundary conditions (15) assure the continuity of the flux j_x as can be verified by a straightforward calculation from the equations (17)

$$j_{xl} = j_{x+} + j_{x-} = j_{xr}.$$

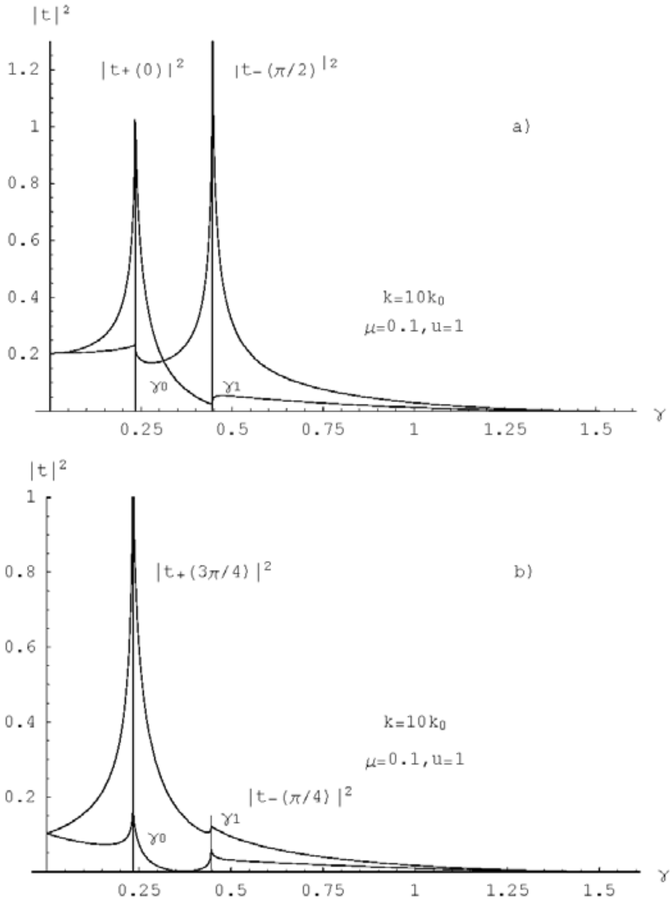

Fig. 4. The squared moduli of the transmitted amplitudes for two couples of orthogonal spin polarizations. The cusps sign the passage through the limit angles. a) The amplitudes $t_+(0)$ and $t_-(\pi/2)$ refer to electrons injected in the R zone by a ferromagnet with a magnetization parallel or antiparallel to the z axis respectively. b) The amplitudes $t_+(3\pi/4)$ and $t_-(\pi/4)$ refer to electrons injected in the R zone by a ferromagnet with a magnetization orthogonal to the interface, that is antiparallel or parallel to the x axis respectively.

When $\gamma < \gamma_0$ both the modes propagate in R zone. When $\gamma_0 < \gamma < \gamma_1$ only the $(-)$ mode remains. The transmission coefficient is the ratio of j_{xr} with the incident flux $j_i = \hbar k \cos\gamma / m_F$,

$$T = \frac{j_{xr}}{j_i},$$

while the reflection coefficient is $R = (j_i - j_{xr})/j_i$:

$$T_+(\delta,\gamma) = (k_+ + k_0)\cos\alpha \cdot |t_+|^2 \vartheta(\gamma_0 - \gamma)/\mu k \cos\gamma$$

$$T_-(\delta,\gamma) = (k_- - k_0)\cos\beta \cdot |t_-|^2 \vartheta(\gamma_1 - \gamma)/\mu k \cos\gamma$$

$$T(\delta,\gamma) = T_+(\delta,\gamma) + T_-(\delta,\gamma)$$

$$R(\delta,\gamma) = |r_\uparrow|^2 + |r_\downarrow|^2 . \qquad (18)$$

When γ overcomes γ_1, $T(\delta,\gamma) = 0$ and $R(\delta,\gamma) = 1$. The flux is conserved because in all the cases

$$T(\delta,\gamma) + R(\delta,\gamma) = 1.$$

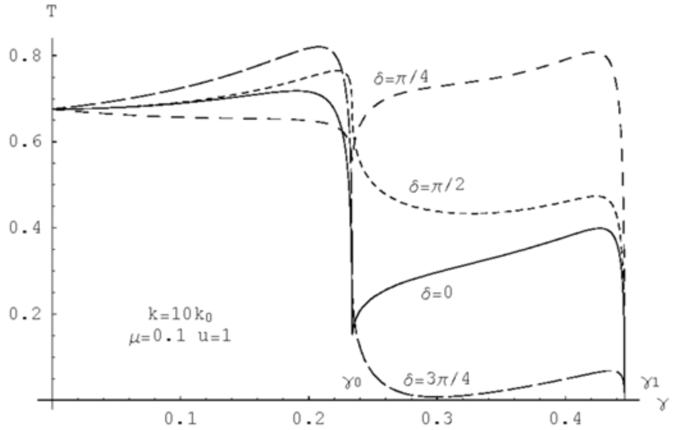

Fig. 5. The two steps of the transmission coefficient T, at $k/k_0 = 10$, for different values of the incoming electron's spin δ. The second step tends to disappear for $\delta = 3\pi/4$ and has the same height of the first one when $\delta = \pi/4$.

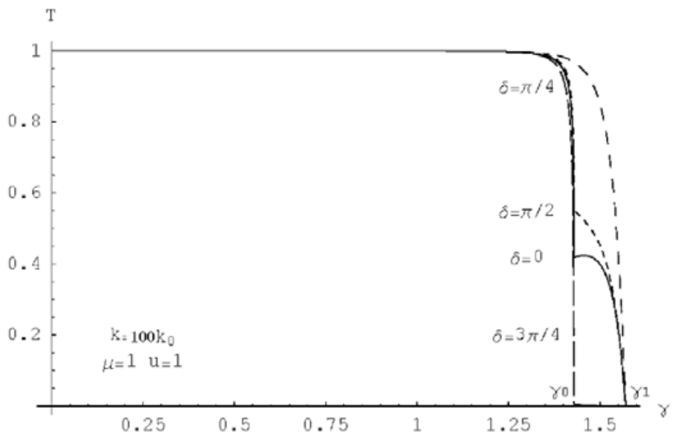

Fig. 6. The two steps of the transmission coefficient T, at $k/k_0 = 100$, for different values of the incoming electron's spin δ. In the limit $k/k_0 \to \infty$, $T = 1$ for $0 < \gamma < \pi/2$.

The transmission coefficient as a function of γ has a first higher step up to γ_0 followed by a lower step that ends in γ_1. Figure 5 shows how the shape and the height of the two steps vary with the spin polarization angle δ. At low values of μ, when the electrons in 2DEG are lighter, the propagation in the $x > 0$ region is allowed at angles nearer to the normal. At equal masses ($\mu = 1$) the passage is allowed up to grazing incidence and the steps appear more squared. We note that the second step tends to disappear around $\delta = 3\pi/4$ and has the maximum height around $\delta = \pi/4$. Figure 6 refers to the case of an higher Fermi wave vector k. Obviously when $k/k_0 \to \infty$, $T = 1$ for γ from 0 to $\pi/2$ but the second step is again present for k greater then k_0 of two magnitude orders.

The transmission of the interface can be analyzed not only in terms of the (+) and (−) modes, but also separating the output probability current in a spin up and a spin down part. The spin up $T_\uparrow(\delta)$ and the spin down $T_\downarrow(\delta)$ transmission coefficients are

$$T_\uparrow(\delta, x) = \frac{1}{\mu k \cos\gamma}\Big\{|t_+|^2 \cos^2\frac{\alpha}{2}(k_+\cos\alpha + k_0) + |t_-|^2 \sin^2\frac{\beta}{2}(k_-\cos\beta + k_0) + |t_+||t_-|\cos\frac{\alpha}{2}\sin\frac{\beta}{2} \times (k_+\cos\alpha + k_-\cos\beta + 2k_0)\cos[(k_+\cos\alpha - k_-\cos\beta)x + \tau_+ - \tau_-]\Big\} \quad (19)$$

$$T_\downarrow(\delta, x) = \frac{1}{\mu k \cos\gamma}\Big\{|t_+|^2 \sin^2\frac{\alpha}{2}(k_+\cos\alpha - k_0) + |t_-|^2 \cos^2\frac{\beta}{2}(k_-\cos\beta - k_0) - |t_+||t_-|\sin\frac{\alpha}{2}\cos\frac{\beta}{2} (k_+\cos\alpha + k_-\cos\beta - 2k_0)\cos[(k_+\cos\alpha - k_-\cos\beta)x + \tau_+ - \tau_-]\Big\}. \quad (20)$$

Here $t_+ = |t_+|\exp(i\tau_+)$, $t_- = |t_-|\exp(i\tau_-)$. Now the precession spin motion gives a spatial modulation of the transmission coefficients $T_\uparrow, T_\downarrow$, while the transmission in (+) and (−) modes, T_+ and T_-, are independent on x. Obviously the oscillations in $T_\uparrow$ and $T_\downarrow$ are opposite in phase and

$$T_\uparrow + T_\downarrow = T_+ + T_- = T.$$

Figure 7a shows the oscillations of $T_\uparrow$ and $T_\downarrow$ when the incidence angle is below the first limit angle γ_0 and the spin in entrance is down. Figure 7b shows what happens when the incidence angle is above γ_0. Near to the interface the contribution of the evanescent waves of mode + appears and far from $x = 0$ we have $T_\uparrow + T_\downarrow = T_-$ and all the transmission coefficients are independent on x. The incoming spin is up and far from the interface the transmitted spin is mostly down: at a distance large enough, the interface is able to turn down the spin.

We note that an incidence out of the normal, with $\gamma \neq 0$, has been recently discussed for small incidence angles [17,18]. Here we have presented a full analysis at any incidence angle focusing the attention on the birefringence.

The previous analysis applies when the incoming wave function is in the *pure* spin state $|\delta\rangle$. Now we take into account the scattering when the incoming electronic spin is in the unpolarized *statistical mixture*. When the electron is unpolarized, its state is not completely known and we represent it with the density operator [19]

$$\rho_U = \frac{1}{2}|\uparrow\rangle\langle\uparrow| + \frac{1}{2}|\downarrow\rangle\langle\downarrow| \quad (21)$$

while the density operator of the completely known pure state is

$$\rho_P = |\delta\rangle\langle\delta|.$$

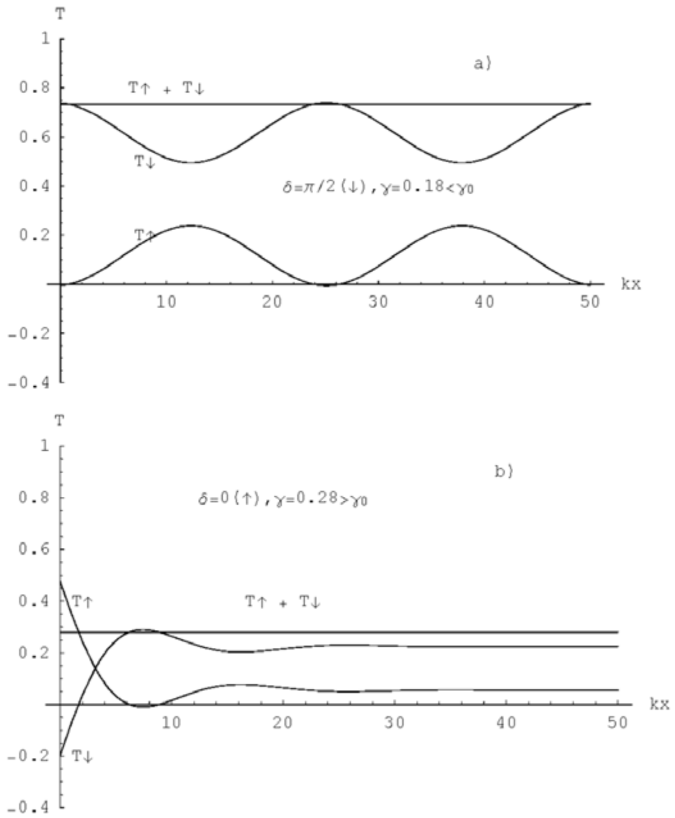

Fig. 7. The transmission coefficients $T_\uparrow$, $T_\downarrow$ for $k/k_0 = 10$, $\mu = 0.1$ and $u = 1$. Figures a and b refer, respectively, to incidence angles γ less than and greater than the first limit angle γ_0.

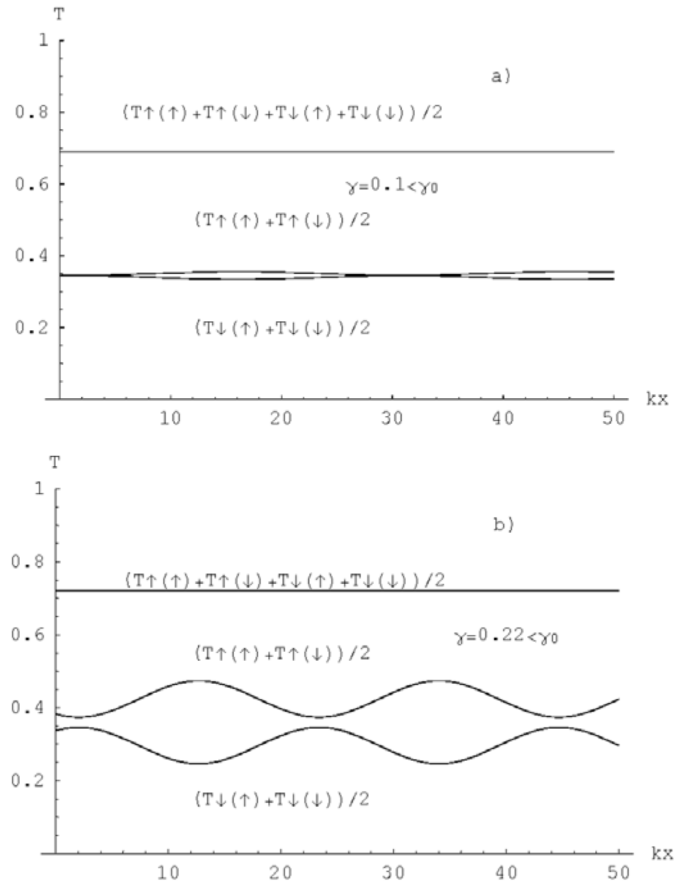

Fig. 8. The transmission coefficients $(T_\uparrow(\uparrow) + T_\uparrow(\downarrow))/2$ and $(T_\downarrow(\uparrow) + T_\downarrow(\downarrow))/2$ for an unpolarized incident wave ($k/k_0 = 10,\ \mu = 0.1,\ u = 1$). Figure a is relative to an almost normal incidence, Figure b refers to γ a little below γ_0.

The mean value of an operator A for the statistical mixture ρ_U (ρ_P) is given by $\langle A\rangle_U = \mathrm{Tr}\ \rho_U A$ ($\langle A\rangle_p = \mathrm{Tr}\ \rho_P A$). We get for the spin components

$$\langle\sigma_x\rangle_P = \sin 2\delta\ ,\ \langle\sigma_y\rangle_P = 0\ ,\ \langle\sigma_z\rangle_P = \cos 2\delta; \quad 0 \le \delta \le \pi$$

in the pure state case and

$$\langle\sigma_x\rangle_U = \langle\sigma_y\rangle_U = \langle\sigma_z\rangle_U = 0$$

for the unpolarized statistical mixture: in this case the mean values of all the spin components of the incident wave function are null. This corresponds to have the injection in the Rashba 2DEG using a simple metallic contact without ferromagnets. The density current probabilities for the incoming spin states $|\uparrow\rangle$ and $|\downarrow\rangle$ sum up in a classical way without quantum interference with equal weights 1/2 and 1/2: all we need are the four transmission coefficients of the equations (19, 20) $T_\uparrow(0)\,, T_\uparrow\left(\frac{\pi}{2}\right), T_\downarrow(0)\,, T_\downarrow\left(\frac{\pi}{2}\right)$. We indicate (0) as ($\uparrow$) and $\left(\frac{\pi}{2}\right)$ as ($\downarrow$) in such a way that the indexes refer to outgoing spins while the incoming spins are shown between the parentheses. We have compared the overall transmission coefficient in spin up state $(T_\uparrow(\uparrow) + T_\uparrow(\downarrow))/2$ with the overall transmission coefficient in spin down state $(T_\downarrow(\uparrow) + T_\downarrow(\downarrow))/2$. Figure 8a shows the transmission of the unpolarized state in up and down spin channels when the incidence is near to the normal, while Figure 8b shows the transmission coefficients at a larger incidence angle γ, a little below the first limit angle. In the first case the interface is not able to discriminate the spins, in the second one it introduces a polarization. The oblique scattering, due to the presence of the birefringence, gives in output a spin up probability current different from spin down probability current.

4 Conductance of a point contact

The foregoing analysis can be applied to describe a point contact device with a semimetallic ferromagnetic source (injecting polarized electrons) or a simply metallic source (injecting unpolarized electrons).

Let a constriction of width W separates the two regions that are connected with two perfect reservoirs at the Fermi energy

$$E_F = \frac{\hbar^2 k^2}{m_F} = E_\pm = \frac{\hbar^2}{2m_S}\left(k'^{\,2} \pm 2k_0 k'\right).$$

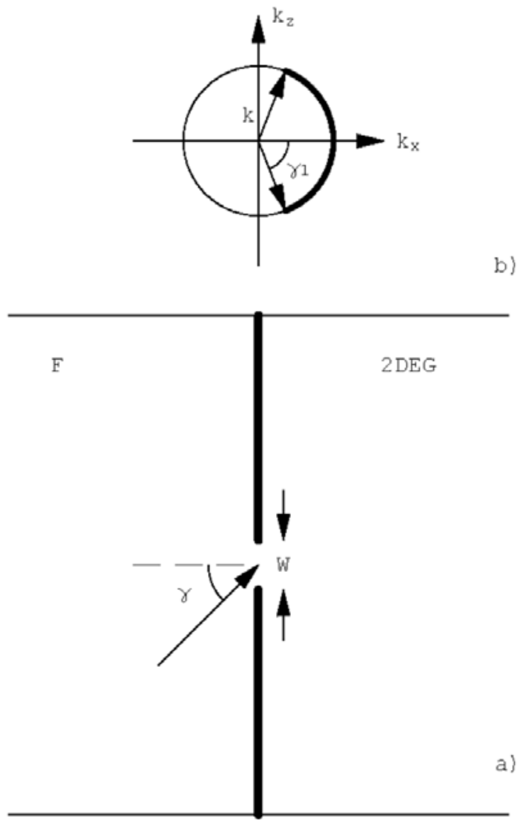

Fig. 9. a) The sketch of the point contact. b) The Fermi circle in k-space. The thick arch indicates the states that carry current into the point contact.

The electron motion within the hybrid system is assumed as ballistic, that is, the electronic mean free path is much longer than the width W of the point contact. The Landauer-Büttiker formalism applies [20,21]. The conductance G at zero temperature is given by

$$G = \frac{e^2}{h} \sum_i T_i, \tag{22}$$

where T_i are the transmission coefficients for all the open channels i between the two reservoirs at the energy E_F. In our case the index i represents the incidence angle γ.

A sketch of the point contact is shown in Figure 9a. The 2D Fermi surface in k-space appears in Figure 9b and only the states on its edge can carry current at zero temperature. As we have shown before, the current is carried through the point contact by the states belonging to the arch from $-\gamma_1$ to γ_1 on the Fermi surface. Quantum mechanically, the current through the point contact is equipartitioned among the $1D$ subbands, or transverse modes, in the constriction. The gap along k_z axis between two consecutive subbands can be estimated of the order of π/W (this is exactly the result for a square well lateral confining potential of width W). The

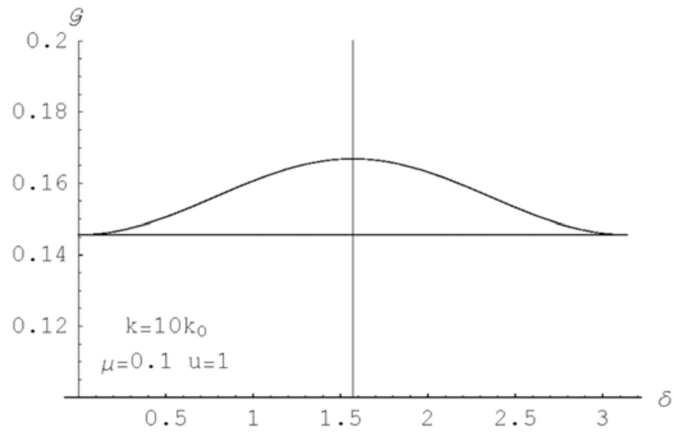

Fig. 10. The conductance for polarized electrons $\mathcal{G}$ as a function of the spin polarization δ of the incoming electrons. The value $\delta = 0$ corresponds to spin up and $\delta = \pi/2$ to spin down.

number of states contained in the element of arch $d\gamma$ is then $kd\gamma/\left(\pi/W\right)$. The equation (22) implies that hybrid system conductance G (with a ferromagnetic source in which the majority carriers are electrons in $|\delta\rangle$ spin state) is

$$G = \frac{e^2}{h} \int_{-\gamma_1}^{\gamma_1} T\left(\delta, \gamma\right) \frac{kW d\gamma}{\pi} = \frac{e^2 kW}{h} \mathcal{G}\left(\delta\right) \tag{23}$$

with

$$\mathcal{G}\left(\delta\right) = \frac{1}{\pi} \int_{-\gamma_1}^{\gamma_1} T\left(\delta, \gamma\right) d\gamma. \tag{24}$$

An exhaustive discussion about this approach can be found in references [21,22]. We note that the restriction around the normal incidence $\gamma = 0$ gives

$$\mathcal{G}_0 = \frac{T\left(\delta, 0\right)}{\pi} d\gamma,$$

i.e. the Sharvin resistance formula [23] used by Grundler [7]. We have seen that T at $\gamma = 0$ is independent on δ and $\mathcal{G}_0$ is independent on the spin polarization.

Figure 10 shows $\mathcal{G}\left(\delta\right)$ for δ between 0 and π. The symmetry relation $T\left(\pi - \delta, -\gamma\right) = T\left(\delta, \gamma\right)$ given in Appendix B gives $\mathcal{G}\left(\pi - \delta\right) = \mathcal{G}\left(\delta\right)$. The conductance of a single interface when the incoming spin is up $(\delta = 0, \pi)$ $\mathcal{G}\left(\uparrow\right)$ is different from the conductance when the incoming spin is down $(\delta = \pi/2)$ $\mathcal{G}\left(\downarrow\right)$. This effect is a direct consequence of the double refraction at the interface that changes the spin state when the electron comes into the R zone. Using the values of parameters of Figure 10 we get for the spin polarization of the conductance

$$\frac{\mathcal{G}\left(\downarrow\right) - \mathcal{G}\left(\uparrow\right)}{\mathcal{G}\left(\downarrow\right) + \mathcal{G}\left(\uparrow\right)} = 0.068.$$

The results for the transmission of an unpolarized beam can be used to discuss the case of a simply metallic

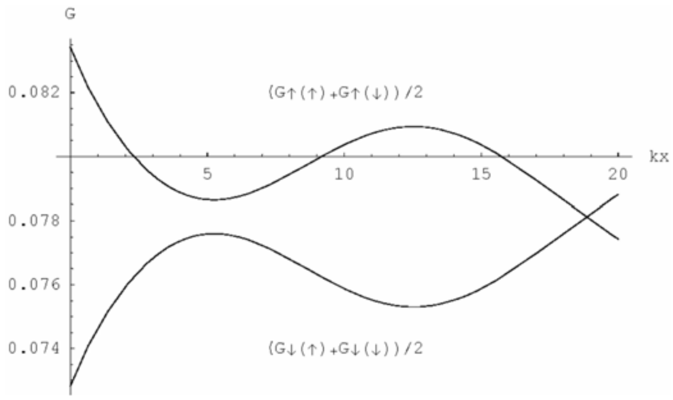

Fig. 11. The conductances for unpolarized electrons $(\mathcal{G}_\uparrow(\uparrow)+\mathcal{G}_\uparrow(\downarrow))/2$ and $(\mathcal{G}_\downarrow(\uparrow)+\mathcal{G}_\downarrow(\downarrow))/2$ as functions of kx. The oscillation period is equal to the precession length π/k_0.

source. It is interesting to notice that the integration upon the angles of incidence does not cancel the effect of partial polarization that the scattering introduces on an unpolarized beam. The conductances for spin up and spin down in the unpolarized case $(\mathcal{G}_\uparrow(\uparrow)+\mathcal{G}_\uparrow(\downarrow))/2$ and $(\mathcal{G}_\downarrow(\uparrow)+\mathcal{G}_\downarrow(\downarrow))/2$ are shown in Figure 11. For the unpolarized case, with the same parameters of Figure 10, we get, at $x=0$

$$\frac{(\mathcal{G}_\uparrow(\uparrow)+\mathcal{G}_\uparrow(\downarrow))-(\mathcal{G}_\downarrow(\uparrow)+\mathcal{G}_\downarrow(\downarrow))}{(\mathcal{G}_\uparrow(\uparrow)+\mathcal{G}_\uparrow(\downarrow))+(\mathcal{G}_\downarrow(\uparrow)+\mathcal{G}_\downarrow(\downarrow))}=0.0362.$$

For unpolarized electrons the spin polarization of the conductance reduces at half of the value for polarized electrons. If we consider the output in spin up channel or in spin down channel, the scattering at a single interface gives rise to a partial polarization. A second interface, acting as an analyzer of the spin, is needed to let the effect be experimentally detectable. We note that it would be worthwhile investigating the effects of a second interface on the spin polarization. We expect that in this case Fabry-Perot oscillations superimpose to those shown in Figure 10 [17,24]. However the study of this problem is currently in progress.

We note that the key ingredient in order to achieve a different transmission of spin up and spin down is the oblique incidence of the electron combined with the strict relation between the propagation direction and spin state (see Eq. (6)). This is in contrast with the case of normal incidence which has been usually considered, but it is in agreement with the results reported for confined structures [25]. Indeed the ballistic spin-transport properties of a quasi-one-dimensional wire with a spin-orbit Rashba interaction in a finite piece of it have been studied with a numerical tight binding model by Mireles et al. [15] finding a kind of spin current polarization different from that we have discussed in this paper. In our opinion the issues obtained with a lateral confinement stem from the transversal localization. The constraint of the electron in a channel can be achieved through the overlap of states with k_z different from zero. The scattering of these components at the interface, as in our case, leads to a different spin up and spin down transmission.

5 Conclusions

We have presented an detailed analysis of the transmission through a semiinfinte 2DEG with Rashba spin orbit coupling extending some previous results [17,18]. An electron in a pure state of spin undergoes a double refraction passing through the interface analogously to what happens to the polarized light at the surface of a biaxial crystal. We have shown that the correct boundary conditions give rise to a spin-dependent transmission coefficient and that the normal incidence is a special case for which the dependence on spin is lost. The results for the scattering of an electron in a pure spin state can be used to investigate the transmission of an unpolarized statistical mixture of spin up and spin down. This analysis shows that a partial spin polarization of an unpolarized incoming beam occurs when the electron goes through the interface in a direction out of the normal. The conductance of a point contact at the interface in ballistic transport regime within the Landauer-Büttiker formalism has been calculated. It has been shown that the conductance for ingoing electrons in a pure spin state has an oscillatory behavior with the polarization angle of the spin. However a partial spin polarization remains also for an unpolarized ingoing beam as the conductances in spin up and spin down channels show.

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Appendix A: Boundary conditions calculation

The boundary conditions at $x=0$ give the following linear system for the amplitudes

$$\begin{aligned}
&t_+\cos\frac{\alpha}{2}+t_-\sin\frac{\beta}{2}-r_+=\cos\delta\\
&-t_+\sin\frac{\alpha}{2}+t_-\cos\frac{\beta}{2}-r_-=\sin\delta\\
&k_+t_+\cos\alpha\cos\frac{\alpha}{2}+k_-t_-\cos\beta\sin\frac{\beta}{2}\\
&\quad+r_+\left(\mu k\cos\gamma+k_0+iu\right)=\cos\delta\left(\mu k\cos\gamma-k_0-iu\right)\\
&-k_+t_+\cos\alpha\sin\frac{\alpha}{2}+k_-t_-\cos\beta\cos\frac{\beta}{2}\\
&\quad+r_-\left(\mu k\cos\gamma-k_0+iu\right)=\sin\delta\left(\mu k\cos\gamma+k_0-iu\right)
\end{aligned}\tag{A.1}$$

whose solution is

$$r_+ = (C_+ A_{--} \cos\alpha/2 + C_- A_{+-} \sin\beta/2)/D$$

$$r_- = (C_- A_{++} \cos\beta/2 + C_+ A_{-+} \sin\alpha/2)/D$$

$$t_+ = [(\cos\delta + r_+)\cos\beta/2 - (\sin\delta + r_-)\sin\beta/2] / \cos\frac{\alpha-\beta}{2}$$

$$t_- = [(\sin\delta + r_-)\cos\alpha/2 + (\cos\delta + r_+)\sin\alpha/2] / \cos\frac{\alpha-\beta}{2} \quad \text{(A.2)}$$

where

$$A_{++} = k_+ \cos\alpha + \mu k \cos\gamma + k_0 + iu$$

$$A_{+-} = k_+ \cos\alpha + \mu k \cos\gamma - k_0 + iu$$

$$A_{-+} = -k_- \cos\beta - \mu k \cos\gamma - k_0 - iu$$

$$A_{--} = k_- \cos\beta + \mu k \cos\gamma - k_0 + iu$$

$$C_+ = (-k_+ \cos\alpha + \mu k \cos\gamma - k_0 - iu)\cos\delta\cos\frac{\beta}{2} - (-k_+ \cos\alpha + \mu k \cos\gamma + k_0 - iu)\sin\delta\sin\frac{\beta}{2}$$

$$C_- = -(k_- \cos\beta - \mu k \cos\gamma + k_0 + iu)\cos\delta\sin\frac{\alpha}{2} + (-k_- \cos\beta + \mu k \cos\gamma + k_0 - iu)\sin\delta\cos\frac{\alpha}{2}$$

$$D = A_{++}A_{--}\cos\frac{\beta}{2}\cos\frac{\alpha}{2} - A_{+-}A_{-+}\sin\frac{\beta}{2}\sin\frac{\alpha}{2}.$$

We note that the amplitudes depend on the spin of the incident wave, represented by the parameter δ, and on the incident wave vector $\vec{k}$. These parameters are not conserved but they characterize the asymptotic wave packet directed towards the surface $x = 0$ before the time at which the scattering starts.

Appendix B: On the symmetry of the system

We begin to consider the symmetry transformation $\widehat{K}_1$ that inverts both p_z and σ_x and leaves p_x and σ_z the same

$$\widehat{K}_1 p_z \widehat{K}_1^\dagger = -p_z \quad ; \quad \widehat{K}_1 \sigma_x \widehat{K}_1^\dagger = -\sigma_x$$

we find that $[H,\ \widehat{K}_1] = 0$. As

$$\alpha(-\gamma) = -\alpha(\gamma) \quad , \quad \beta(-\gamma) = -\beta(\gamma)$$

and changing the incident spin orientation δ in $\pi-\delta$, we get

$$t_+(\pi-\delta,-\gamma) = -t_+^*(\delta,\gamma)$$

$$t_-(\pi-\delta,-\gamma) = t_-(\delta,\gamma)$$

$$r_+(\pi-\delta,-\gamma) = -r_+^*(\delta,\gamma)$$

$$r_-(\pi-\delta,-\gamma) = r_-(\delta,\gamma).$$

The total transmission obeys the equation

$$T(\pi-\delta,-\gamma) = T(\delta,\gamma) \quad \text{(B.1)}$$

and T is an even function of γ only for spin up $(\delta = 0)$ and spin down $(\delta = \pi/2)$ while T is asymmetrical around the normal incidence $\gamma = 0$ for a generic spin state. We note that at $\gamma = 0$ the "partial" time-reversal $\widehat{K}_1$ again does not say anything about the relationship between $T(\delta = 0, \gamma = 0)$ and $T(\delta = \pi/2, \gamma = 0)$.

The Hamiltonian H depends on the electrical field as a parameter through the constant k_0. The inversion of the electrical field is obtained by changing the sign of k_0. The spectrum of $H(k_0)$ can be mapped into the spectrum of $H(-k_0)$ by means of an operator $\widehat{K}_2$ that inverts $\vec{\sigma}$ that is

$$\widehat{K}_2 \vec{\sigma} \widehat{K}_2^\dagger = -\vec{\sigma}.$$

The operator $\widehat{K}_2$ transforms all the spinors into orthogonal spin states and interchanges the $(+)$ mode with the $(-)$ mode. We find

$$k_\pm(k_0) = k_\mp(-k_0)$$

$$\alpha(-k_0) = \beta(k_0)$$

and

$$t_+(k_0,\delta) = t_-(-k_0,\delta+\pi/2)$$

$$t_-(k_0,\delta) = -t_+(-k_0,\delta+\pi/2)$$

$$r_+(k_0,\delta) = r_-(-k_0,\delta+\pi/2)$$

$$r_-(k_0,\delta) = -r_+(-k_0,\delta+\pi/2).$$

The total transmission has the form

$$T = \frac{1}{\mu}\cdot\frac{k_0}{k}\cdot\frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}\cdot\frac{|t_+|^2\cos\alpha + |t_-|^2\cos\beta}{\cos\gamma}$$

equivalent to that of equation (18) from it follows that

$$T(-k_0,\delta+\pi/2) = T(k_0,\delta). \quad \text{(B.2)}$$

Equation (B.2) sets up a relation between the spin up and spin down state at any incidence angle when the electrical field is inverted but again it is not able to give the equality of $T_+(\uparrow)$ and $T_-(\downarrow)$ when $\gamma = 0$ at the same value of the parameter k_0.

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